Solutions

Final Exam Review
Chapters A-D and 1-4

Simplify using exponent rules.

1. $\left(\frac{8x^3y^{-6}}{x^{-8}y^{-4}}\right)^{1/3}$

$$8^{1/3}\left(\frac{x^3}{x^{-8}}\right)^{1/3}\left(\frac{y^{-6}}{y^{-4}}\right)^{1/3} = 2\left(x^{11}\right)^{1/3}\left(\frac{1}{y^2}\right)^{1/3}$$

$$= 2\,\frac{x^{11/3}}{y^{2/3}}$$

2. $\sqrt[3]{8x^3y^6}\,\sqrt[4]{2x^5y}$

$$2xy^2 \cdot 2^{1/4}x^{5/4}y^{1/4} = 2^{5/4}x^{9/4}y^{9/4}$$

3. $\sqrt{x^2\sqrt[3]{x^4}}$

$$\left(x^2\left(x^4\right)^{1/3}\right)^{1/2} = \left(x^2\cdot x^{4/3}\right)^{1/2} = \left(x^{10/3}\right)^{1/2} = x^{10/6} = x^{5/3}$$

Perform the indicated operation and simplify.

4. $\frac{x^2-10x+21}{2x^2-12x-14} \div \frac{x^2+2x-15}{2x^2+12x+10}$

$$\frac{(x-3)(x-7)}{2(x-7)(x+1)} \cdot \frac{2(x+5)(x+1)}{(x+5)(x-3)} = 1$$

5. $\frac{x}{x^2+2x+1} - \frac{1}{x+1}$

$$\frac{x}{(x+1)^2} - \frac{1}{x+1} = \frac{x}{(x+1)^2} - \frac{(x+1)}{(x+1)^2} = \frac{-1}{(x+1)^2}$$

Solve the inequality. Write your solution in interval notation and graph it on the real number line.

6. $x^2 - 5x - 6 > 0$

$(x-6)(x+1) > 0$

+ + + + + | − − − − − | + + + + +

−1   6

$x \in (-\infty, -1) \cup (6, \infty)$

7. $-3 \geq -8 - 5x > -27$

$+8 \quad +8 \qquad +8$

$5 \geq -5x > 19$

$\boxed{-1 \leq x < \frac{19}{5}}$ $\boxed{x \in [-1, \frac{19}{5}).}$

Solve the quadratic equation by factoring.

8. $x^2 + 7x = 30$

$x^2 + 7x - 30 = 0$

$(x+10)(x-3) = 0 \qquad x = -10 \text{ or } 3.$

Solve the quadratic equation by any method learned in class.

9. $x^2 + 7x + 1 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-7 \pm \sqrt{45}}{2} = \frac{-7 + \sqrt{45}}{2} \text{ or } \frac{-7 - \sqrt{45}}{2}$$

Factor completely.

10. $y^2(x^2-4)-(x^2-4)$

$(y^2-1)(x^2-4) = (y+1)(y-1)(x+2)(x-2)$

11. $4w^2+4wy+y^2$

$(2w+y)^2$

12. $5x^3+10x^2-2x-4$

$5x^2(x+1)-2(x+1) = (5x^2-2)(x+1)$

13. $4(2x+1)^2-9$

$(2(2x+1)-3)^2 = (4x-1)^2$

14. Let $P(8,4)$ and $Q(6,-2)$ be two points in the coordinate plane.

(a) Find the distance between the points P and Q.

$\sqrt{(4-(-2))^2+(8-6)^2} = \sqrt{6^2+2^2} = \sqrt{40}$

(b) Find the midpoint between the points P and Q.

$\frac{8+6}{2} = 7$ $(7,1)$

$\frac{4+(-2)}{2} = 1$

15. A set of data is given in the following table. Find an equation to model the data. Use your model to predict the value of y when $x = 20$.

x	y
0	12
1	17
2	22
3	27
4	32

Linear

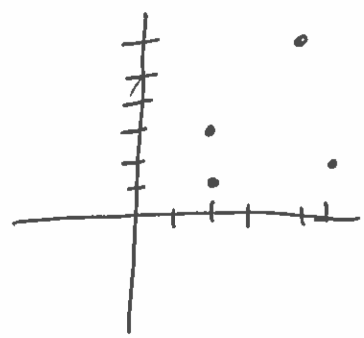
$$Y = 5x + 12$$

So $y = 5(20) + 12 = 112$ when $x = 20$.

16. A set of ordered pairs defining a relation is given below.

$$\{(5, 2), (4, 6), (2, 3), (2, 1)\}$$

(a) Find the domain of the relation.
(b) Find the range of the relation.
(c) Sketch a diagram of the relation.
(d) Does the relation define a function?



Dom $= \{5, 4, 2\}$

Ran $= \{2, 6, 3, 1\}$

Not a function.
Input 2 has multiple outputs.

17. Consider the function given by

$$r(z) = \frac{8(z-4)^2}{z-3}.$$

(a) What is the name of the function?
(b) What letter represents the input?
(c) What is the output?
(d) Find r(3). What does it represent?
(e) What is the domain of the function?

(a) r
(b) z
(c) $r(z)$ or $\frac{8(z-4)^2}{z-3}$
(d) $r(3) =$ undefined.
(e) $\mathbb{R} \setminus 3$. All but 3.

18. When a skydiver jumps out of an airplane from a height of 13,000 ft, her height h above the ground after t seconds is given by the function

$$h(t) = 13,000 - 16t^2.$$

(a) Find $h(10)$ and $h(20)$. What do these values represent?

(b) For safety reasons a sky diver must open the parachute at a height of about 2500 ft (or higher). A sky diver opens her parachute after 24 seconds. Did she open the parachute at a safe height?

(c) Find the net change in the sky diver's height from 0 to 25 seconds.

(a) $h(10) = 13{,}000 - 16(10)^2 = 11{,}400$ ft above ground after 10 seconds

$h(20) = 13{,}000 - 16(20)^2 = 6{,}600$ ft above ground after 20 sec.

(b) $h(24) = 3{,}784$ ft, yes.

(c) $h(25) - h(0) = 3{,}000 - 13{,}000 = -10{,}000$ ft

Find the domain, inverse and range.

19. $f(x) = \frac{1}{\sqrt{x-3}}$. $\mathrm{Dom}(f) = (3, \infty)$

$x = \frac{1}{\sqrt{y-3}}$

$\sqrt{y-3} = \frac{1}{x} \Rightarrow y - 3 = \frac{1}{x^2} \Rightarrow y = \frac{1}{x^2} + 3 = f^{-1}(x)$ $\mathrm{Dom}(f^{-1}) = (0, \infty) = \mathrm{Ran}(f)$

20. $f(x) = \ln(\frac{x}{3})$.

$\mathrm{Dom}(f) = (0, \infty)$

$x = \ln(\frac{y}{3})$

$e^x = y/3 \Rightarrow y = 3e^x = f^{-1}(x)$ $\mathrm{Dom}(f^{-1}) = (-\infty, \infty) = \mathrm{Ran}(f)$.

21. Find a function that models the number of q quarters in D dollars.

4 quarters in a dollar

$D = 4q$

Combine into a single logarithm.

22. $3\log_5(x) - 4\log_5(x) + 8\log_5(y)$

$$\log_5(x^3) - \log_5(x^4) + \log_5(y^8) = \log_5\left(\frac{x^3}{x^4} \cdot y^8\right) = \log_5\left(\frac{y^8}{x}\right)$$

23. $1/2\ln(2) + 1/4\ln(z) + 1/8\ln(y)$

$$\ln(\sqrt{2}) + \ln(\sqrt[4]{z}) + \ln(\sqrt[8]{y}) = \ln(\sqrt{2} \cdot \sqrt[4]{z} \cdot \sqrt[8]{y})$$

$$\left(= \ln\left(\sqrt{2\sqrt{z\sqrt{y}}}\right)\right).$$

Expand into as many logarithms as possible. (In both cases this is three logarithms)

24. $\log(\frac{z^3}{\sqrt{xy}})$

$$\log(z^3) - \log(\sqrt{xy}) = 3\log(z) - \frac{1}{2}\log(xy) = 3\log z - \frac{1}{2}\log(x) - \frac{1}{2}\log(y)$$

25. $\ln(\frac{a^2-b^2}{c^2})$

$$\ln\left(\frac{(a+b)(a-b)}{c^2}\right) = \ln(a+b) + \ln(a-b) - 2\ln(c).$$

Evaluate without using a calculator.

26. $\log_8 1/2$

$2^3 = 8 \Leftrightarrow 8 = 2^{1/3}$ $\quad \log_8(1/2) = -1/3$

27. $\log_3(27)$

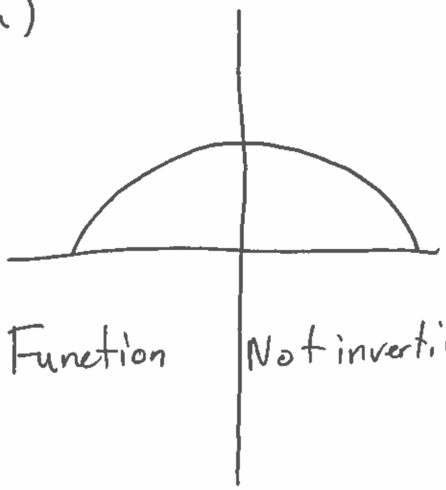
$$\log_3(27) = \log_3(3^3) = 3$$

28. $\log(1,000,000)$

$$\log(1,000,000) = \log(10^6) = 6$$

Determine if the following graphs are functions. If so, determine if they are invertible.

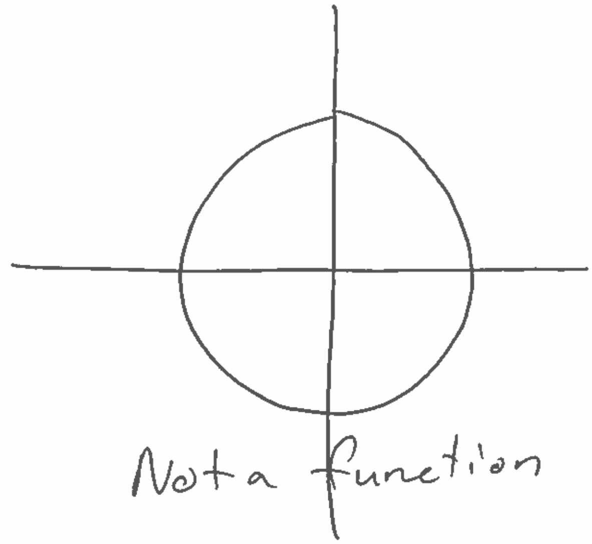
(a)



Function

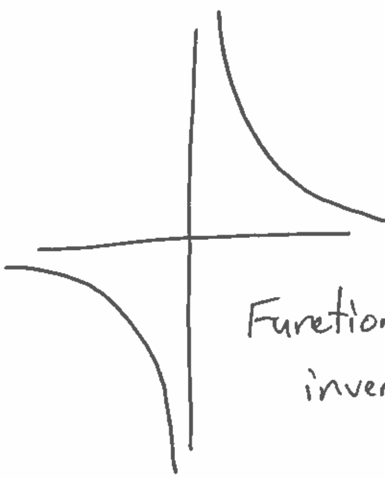
Not invertible

(b)



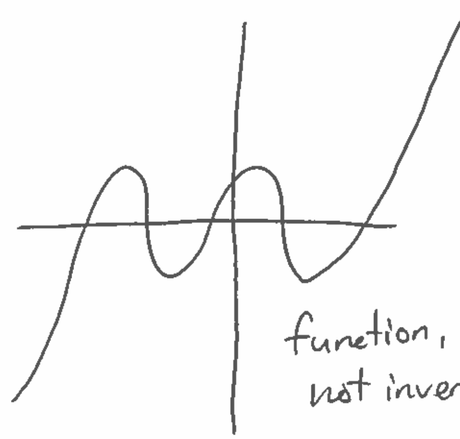
Not a function

(c)



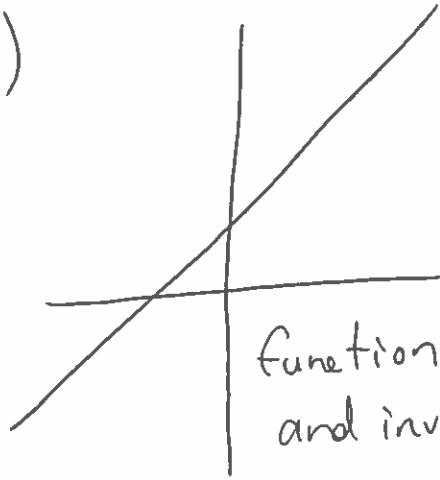
Function and invertible

(d)



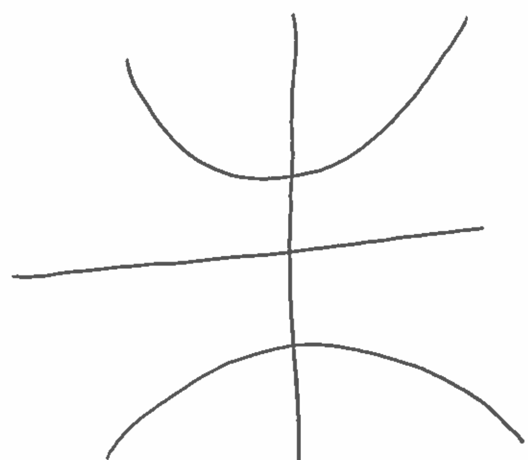
function, not invertible

(e)



function and invertible

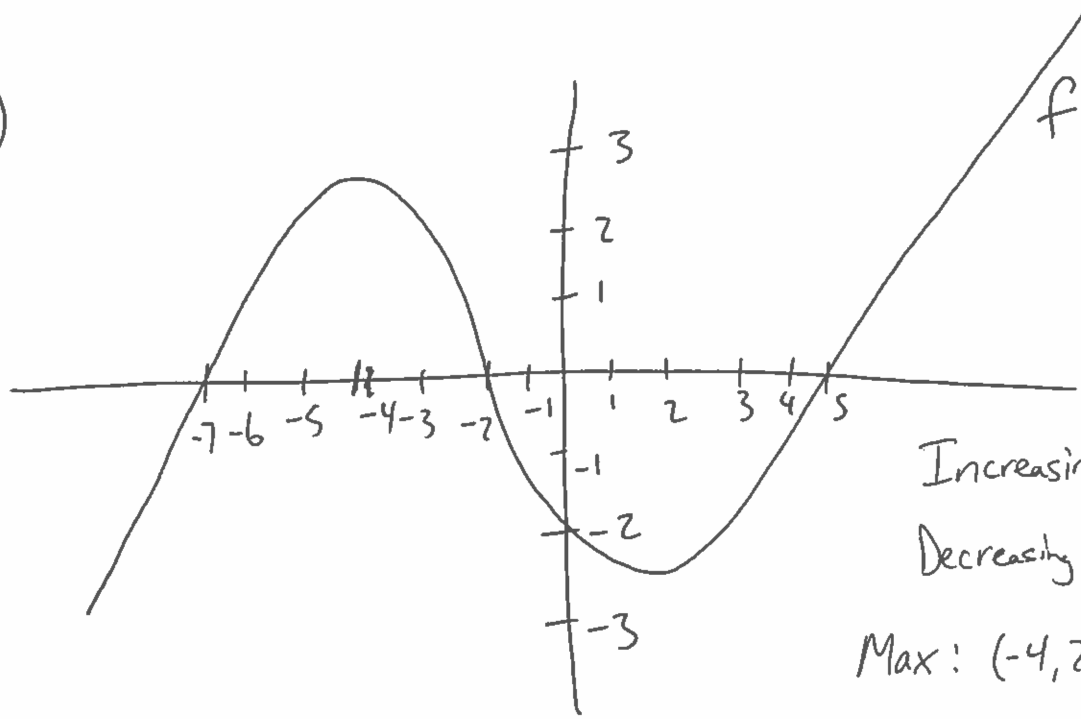
(f)



Not a function

Determine where the function is increasing/decreasing. Find the maxima and minima for the function and identify the x-values corresponding to each.

(a)



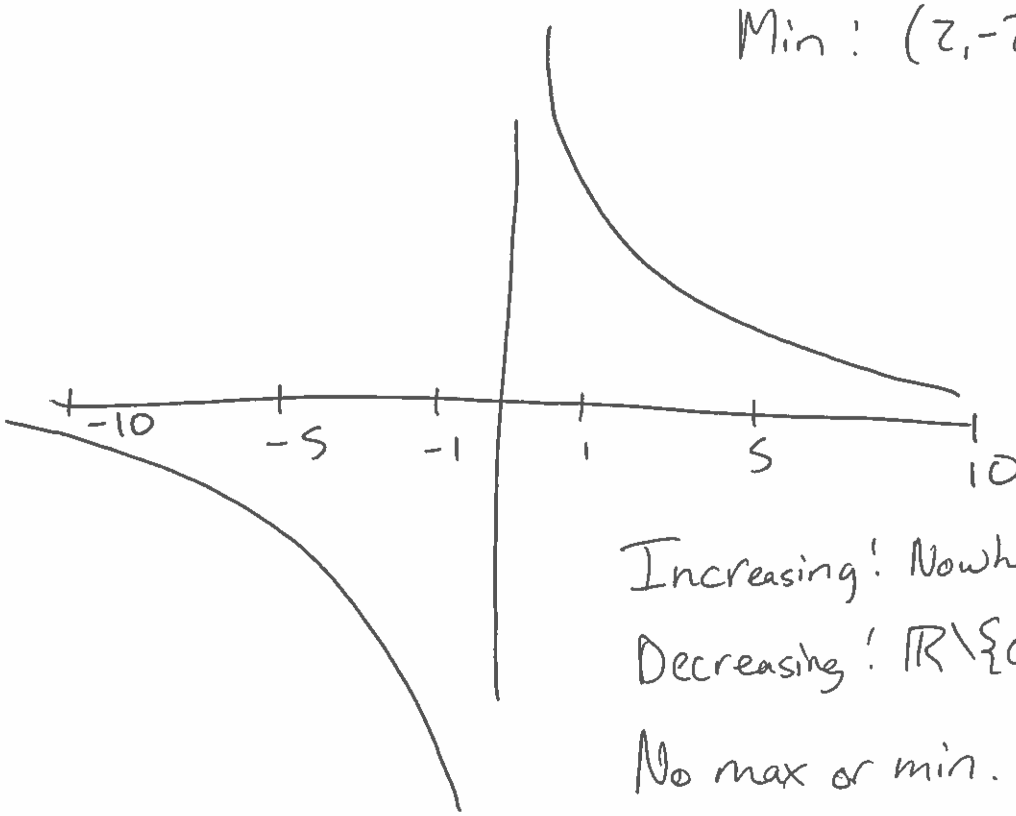
Increasing: $(-\infty,-4] \cup [2,\infty)$

Decreasing: $[-4,2]$

Max: $(-4, 2.5)$

Min: $(2, -2.5)$

(b)



Increasing: Nowhere

Decreasing: $\mathbb{R} \setminus \{0\}$

No max or min.